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| 7MMA2C2 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Second Semester**

**Mathematics**

**ANALYSIS - II**

**(CBCS – 2017 onwards)**

Time : 3 Hours — Maximum : 75 Marks

**Part A** (10 × 2 = 20)

Answer **all** questions.

1. If $f \in \mathfrak{R}$, then prove that $\underline{\int_a^b} f\, d\alpha \leq \overline{\int_a^b} f\, d\alpha$.

2. State fundamental theorem of calculus.

3. Define uniformly convergent sequence.

4. Define a pointwise bounded function on a set $E$.

5. State Taylor's theorem.

6. Prove that the function $E$ is periodic with period $2\pi$.

7. Define outer measure of a set.

8. If $m^* E = 0$, then prove that $E$ is measurable.

9. Define simple function.

10. State Fatou's Lemma.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If $P^*$ is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(P, f, \alpha)$.

Or

(b) If $f$ is monotonic on $[a, b]$ and if $\alpha$ is continuous on $[a, b]$, then prove that $f \in \mathfrak{R}(\alpha)$

12. (a) State and prove the Cauchy criterion for uniform convergence.

Or

(b) Let $\alpha$ be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathfrak{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, 3...$ and if $f_n \to f$ uniformly on $[a, b]$ then prove that $f \in \mathfrak{R}(\alpha)$ on $[a, b]$ and $\int_a^b f\,d\alpha = \lim_{n\to\infty} \int_a^b f_n\,d\alpha$.

13. (a) Suppose $\sum C_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} C_n x^n$, $-1 < x < 1$. Then prove that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} C_n$.

Or

(b) If $x > 0$ and $y > 0$, then prove that $\int_0^1 t^{x-1}(1-t)^{y-1}dt = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

14. (a) Let $\langle E_i \rangle$ be a sequence of measurable sets. Then prove that $m(UE_i) \le \sum mE_i$ .

Or

(b) If $f$ and $g$ be two measurable real valued functions defined on the same domain. Then prove that the functions $f+g$ and $fg$ are also measurable.

15. (a) Let $\varphi$ and $\psi$ be simple functions which vanish outside a uf of finite measure. Then prove that $\int (a\varphi + b\psi) = a\int \varphi + b\int \psi$ and if $\varphi \ge \psi$ a.e., then prove that $\int \varphi \ge \int \psi$ .

Or

(b) State and prove bounded convergence theorem.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Assume α increases menotonically and $\alpha' \in \Re$ on $[a,b]$. Let $f$ be a bounded real function on $[a,b]$. Then prove that $f \in \Re(\alpha)$ if and only if $f\alpha' \in \Re$ . Also prove that $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)dx$ .

17. Suppose $f_n \to f$ uniformly on a set $E$ in a metric space. Let $x$ be a limit point of $E$ and suppose that $\lim_{t\to x} f_n(t) = A_n$ , $n = 1, 2, 3,....$ then prove that $\{A_n\}$ converges and $\lim_{t\to x} f(x) = \lim_{n\to\alpha} A_n$ .

18. State and prove Parseval's theorem.

19. Prove that the outer measure of an interval is its length.

20. Let $f$ be defined and bounded on a measurable set $E$ with $mE$ finite. Prove that the necessary and sufficient condition for $f$ to be measurable is $\inf_{f \le \psi} \int_E \psi(x)dx = \sup_{f \ge \varphi} \int_E \varphi(x)dx$ for all simple functions $\varphi$ and $\psi$.

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| 7MMA2C3 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.**

**Second Semester**

**Mathematics**

**PARTIAL DIFFERENTIAL EQUATIONS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. State existence and uniqueness of solutions of equations of the type $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

2. Define Pfaffian differential equation.

3. Form a partial differential equation by eliminating the arbitrary constants from $z = (x+a)(x+b)$.

4. Define general integral of the equation $F(x, y, z, p, q) = 0$.

5. Find the complete integral of the equation $pq = 1$.

6. Show that the equations $f(x, y, p, q) = 0$, $g(x, y, p, q) = 0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0$.

7. Find the particular integral of the equation $(D^2 - D')z = e^{x+y}$.

8. Verify that the partial differential equation $\frac{\partial^z z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{\partial z}{x}$.

9. Define exterior Neumann problem.

10. Write down the d'Alembert's solution of the one-dimensional wave equation.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$.

Or

(b) Show that the necessary and sufficient condition that the pfaffian differential equation $X.dr = 0$ should be integrable in that $X.curl X = 0$.

12. (a) Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$

Or

(b) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2,\ z = 0$.

13. (a) Find the complete integral of the equation $p^2 x + q^2 y = z.$

Or

(b) Solve $(p^2 + q^2)y = qz$ by Jacobi's method.

14. (a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

15. (a) Find the temperature in a sphere of a radius a when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$.

Or

(b) If $\rho > 0$ and $\psi(r) = \int_S \frac{\sigma(r')ds'}{|r - r'|}$, where the volume $V$ is bounded, prove that $\lim_{r=\infty} r\psi(r) = M$ where $M = \int_V \rho(r')d\tau'$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Verify that the equation $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ is integrable and find its primitive.

17. Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the $x$-axis.

18. Prove that an equation of the "Clairaut" form

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=f\left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right)$$ is always soluble by Jacobi's method.

19. Derive the solution of the equation:

$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}=0$$

for the region $r \geq 0, z \geq 0$, satisfying the conditions :

(a) $v \to 0,$ as $z \to \infty$ and as $r \to \infty$

(b) $v = f(r)$ on $z = 0, r \geq 0.$

20. If the string is released from rest in the position $y=\frac{4\in}{l^2}x(l-x)$. Show that its motion is described by the equation $y=\frac{32\in}{3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\sin\frac{(2n+1)\pi x}{l}\cos\frac{(2n+1)\pi ct}{l}$.

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| F–8457 |
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| 7MMA2E1 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Second Semester**

**Mathematics**

**Elective – GRAPH THEORY**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a spanning sub graph of G. Give an example.

2. Draw all the trees with six vertices.

3. What is meant by the k-connected graph? Give an example for a 3-connected graph.

4. Draw the Herschel graph.

5. Define a perfect matching with an example.

6. Draw 3-edge chromatic graph.

7. What is meant by independence number of G? Give an example.

8. Define the Ramsey numbers. Also find $r(k, 2)$.

9. Draw the embedding of $k_{3,3}$ on the Mobius band.

10. If G is a simple planar graph with $\upsilon \geq 3$, then prove that $\varepsilon \leq 3\upsilon - 6$.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Enumerate the following terms an illustration for each:

    (i) Incidence matrix of graph G;

    (ii) Adjacency matrix of graph G.

Or

(b) Define a spanning tree of G. Also prove that every connected graph contains a spanning tree.

12. (a) With the usual notations, prove that $k \leq k' \leq \delta$.

Or

(b) Prove that $C(G)$ is well defined.

13. (a) State and prove that Hall's theorem.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has 2-edge colouring in which both colours are represented at each vertex of degree at least two.

14. (a) With the usual notations, prove that $\alpha' + \beta' = v$ if $\delta > 0$.

Or

(b) Prove that every critical graph is a block.

15. (a) Let $v$ be a vertex of a planar graph $G$. Prove that $G$ can be embedded in the plane in such a way that $v$ is on the exterior face of the embedding.

Or

(b) State and prove the Euler's formula for plane graph.

## Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an edge $e$ of $G$ is a cut edge of $G$ if and only if $e$ is contained in no cycle of $G$.

17. Show that a non empty connected graph is Eulerian if and only if it has no vertices of odd degree.

18. State and prove that the vizing's theorem.

19. With the usual notations, prove that $r(k,k) \geq 2^{k/2}$.

20. Prove that every planar graph is 5-vertex- colourable.

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.**

**Third Semester**

**Mathematics**

**COMPLEX ANALYSIS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. State the Hadamard's formula.

2. Distinguish between translation, rotation and inversion.

3. Compute $\int_{|z|=2} \frac{dz}{z^2+1}$.

4. State the Morera's theorem.

5. Define an essential isolated singularity. Give an example.

6. Find the poles of $\cot z$.

7. Obtain the residue for $\frac{e^z}{(z-a)(z-b)}$.

8. State the argument principle theorem.

9. Obtain the series expansion for arc $\tan z$ and arc $\sin z$.

10. Define an entire function. Give an example.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the Lucas's theorem.

Or

(b) Derive the complex form of the Cauchy-Riemann equations.

12. (a) Prove that the line integral $\int_{\gamma} pdx + qdy$, defined in $\Omega$, depends only on the end points of $\gamma$ if and only if there exists a function $U(x,y)$ in $\Omega$ with the partial derivatives $\partial u / \partial x = p$, $\partial u / \partial y = q$.

Or

(b) State and prove the Cauchy's estimate theorem. Also deduce that Liouville's theorem.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove the Schwarz lemma.

14. (a) State and prove the Rouche's theorem.

Or

(b) Evaluate $\int_0^{\pi} \frac{d\theta}{a + \cos\theta}, a > 1$.

15. (a) With the usual notations, prove that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$.

Or

(b) Derive the Jensen's formula.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the cross ratio $(z_1, z_2, z_3, z_4)$ is real if and only if the four points lie on a circle or on a straight line.

17. State and prove the Cauchy's representation formula. Deduce that $f^{(n)}(z)=\frac{n!}{2\pi i}\int_C \frac{f(r)dr}{(r-z)^{n+1}}$.

18. State and prove the local mapping theorem.

19. Show that $\int_0^{\pi} \log \sin x\, dx = \pi \log\left(\frac{1}{2}\right)$.

20. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} A_n (z-a)^n$ for the function $f(z)$ analytic $R_1 < |z-a| < R_2$.

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| 7MMA3C2 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Third Semester**

**Mathematics**

**TOPOLOGY – I**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Let $X$ be a set; let $B$ be a basis for a topology $J$ on $X$. Then show that $J$ equals the collection of all unions of elements of $B$.

2. Define the order topology.

3. Find a functions $f : R \to R$ that is continuous at precisely one point.

4. State the sequence lemma.

5. Define locally path connected space.

6. Let A be a connected subset of X. If $A \subset B \subset \overline{A}$, then prove that B is also connected.

7. State the tube lemma.

8. Show that every closed interval in $R$ is compact.

9. Give an example that the product two Lindelöf spaces need not be Lindelöf.

10. Show that a closed subspace of a normal space is normal.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If $A$ is a subspace of $X$ and $B$ is a subspace of $Y$, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

Or

(b) Show that the lower limit topology $J'$ on $R$ is strictly finer than the standard topology $J$.

12. (a) State and prove pasting lemma.

Or

(b) Prove that the topologies on $R^n$ induced by the Euclidean metric d and the square metric $\rho$ are the same as the product topology on $R^n$.

13. (a) Show that $X$ is locally connected if and only if for every open set $U$ of $X$, each component of $U$ is open in $X$.

Or

(b) Let X be an ordered set in the order topology. Show that if X is connected, then X is a linear continuum.

14. (a) Show that every compact subset of a Hausdorff space is closed.

Or

(b) State and prove uniform continuity theorem.

15. (a) Prove that every metrizable space is normal.

Or

(b) Show that every locally compact Hausdorff space is regular.

**Part C** $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let $Y$ be a subspace of $X$; let $A$ be a subset of $Y$; let $\overline{A}$ denote the closure of $A$ in $X$. Then prove that the closure of $A$ in $Y$ equals $\overline{A} \cap Y$.

(b) Show that the collection of open rays in an ordered set $A$ is a subbasis for the order topology on $A$.

17. (a) Let $Y$ be a subspace of $X$. Then prove that a set $A$ is closed in $Y$ if and only if it equals the intersection of a closed set of X with $Y$.

(b) Show that $R \times R$ in the dictionary order topology is metrizable.

18. If $L$ is a linear continuum in the order topology, then show that $L$ is connected and so is every interval and ray in $L$.

19. Let $X$ be a simply ordered set having the least upper bound property. Prove that, in the order topology, each closed interval in $X$ is compact.

20. State and prove Urysohn Metrization theorem.

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| 7MMA3C3 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Third Semester**

**Mathematics**

**PROBABILITY AND STATISTICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. If the sample space is $C = C_1 \cup C_2$ and if $P(C_1) = 0.8$ and $P(C_2) = 0.5$, find $P(C_1 \cap C_2)$.

2. Let $X$ have the p.d.f $f(x) = \begin{cases} \frac{x}{6}, & x \geq 1, 2, 3 \\ 0, & elsewhere \end{cases}$. Then find $E(x^3)$.

3. Define correlation coefficient.

4. Show that the random variables $x_1$ and $x_2$ with joint p.d.f. $f(x_1, x_2) = 12x_1 x_2 (1 - x_2), 0 < x_1 < 1, 0 < x_2 < 1$, zero, elsewhere, are independent.

5. If X is $b(n, p)$, show that $E\left(\frac{X}{n}\right) = P$.

6. If $(1-2t)^{-6}$, $t < \frac{1}{2}$, is the m.g.f. of the random variable X, find $\Pr(X < 5.23)$.

7. Let $X$ have the p.d.f. $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3 \ldots\ldots$, zero, elsewhere. Find the p.d.f. of $Y = x^3$.

8. Find the mean of the beta distribution.

9. If $Y$ is $b\left(100, \frac{1}{2}\right)$, approximate the value of $\Pr(Y = 50)$.

10. Let $\overline{X}_n$ denote the mean of a random sample of size $n$ from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $f(x) = x/15$, $x = 1, 2, 3, 4, 5$, zero elsewhere, be the p.d.f. of $X$. Find $\Pr(X = 1 \text{ } or \text{ } 2)$, $\Pr\left(\frac{1}{2} < x < \frac{5}{2}\right)$, and $\Pr(1 \le x \le 2)$.

Or

(b) Consider the distribution $F(x) = 1 - e^{-x} - xe^{-x}$, $0 \le x < \infty$, zero elsewhere. Find the p.d.f., the mode and the median.

12. (a) Let $f(x_1, x_2) = 21x_1^2 x_2^3$, $0 < x_1 < x_2 < 1$, zero elsewhere, be the joint p.d.f of $x_1$ and $x_2$.

(i) Find the conditional mean and variance of $x_1$, given $x_2 = x_2$, $0 < x_2 < 1$.

(ii) Find the distribution of $Y = E\left( x_1 / x_2 \right)$.

Or

(b) If $x_1$ and $x_2$ are independent random variables with Marginal probability density functions $f_1(x_1)$ and $f_2(x_2)$, respectively, then prove that

$\Pr(a < x_1 < b, C < x_2 < d) = \Pr(a < x_1 < b)\Pr(C < x_2 < d)$. For every $a < b$ and $C < d$, where $a, b, c$ and $d$ are constants.

13. (a) Show that $\int_{\mu}^{\infty} \frac{1}{\Gamma(k)} \quad z^{k-1} e^{-z}\, dz = \sum_{x=0}^{k-1} \frac{\mu^x e^{-\mu}}{x!}$

$k = 1, 2, 3.....$

Or

(b) Two people toss a coin five independent times each. Find the probability that they will obtain the same number of heads.

14. (a) Let X have the uniform distribution over the interval $\left(-\pi/2, \pi/2\right)$. Show that $Y = \tan X$ has a Cauchy distribution.

Or

(b) If $\overline{X}$ is the mean of a random sample of size $n$ from a normal distribution with mean $\mu$ and variance 100, find $n$ so that $\Pr\left(\mu - 5 < \overline{X} < \mu + 5\right) = 0.954$.

15. (a) Let $F_n(Y)$ denote the distribution function of a random variable $Y_n$ whose distribution depends upon the positive integer $n$. Let $C$ denote a constant which does not depend upon $n$. Prove that the sequence $Y_n, n = 1, 2, 3.....$ converges in probability to the constant $C$ if and only if the limiting distribution of $Y_n$ is degenerate at $Y = C$.

Or

(b) Let $\overline{X}_n$ denote the mean of a random sample of size $n$ from a gamma distribution with parameters $\alpha = \mu > 0$ and $\beta = 1$. Show that the limiting distribution of $\sqrt{n}\left(\overline{X}_n - \mu\right) / \sqrt{\overline{X}_n}$ is $N(0, 1)$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove Chebyshev's inequality.

(b) Let $X$ equal the number of heads in four independent flips of a coin. Using certain assumption, determine the p.d.f. of $X$ and compute the probability that $X$ is equal to an odd number.

17. Let $X, Y, Z$ have joint p.d.f. $f(x, y, z,) = 2(x+y+z)/3$, $0 < x < 1$, $0 < y < 1$, $0 < z < 1$, zero elsewhere.

(a) Find the marginal probability density functions.

(b) Compute $\Pr\left(0 < X < \frac{1}{2}, 0 < Y < \frac{1}{2}, 0 < Z < \frac{1}{2}\right)$, and

$\Pr\left(0 < X < \frac{1}{2}\right) = \Pr\left(0 < y < \frac{1}{2}\right) = \Pr\left(0 < z < \frac{1}{2}\right)$.

(c) Are $X$, $Y$ and $Z$ independent?

(d) Calculate $E\left(X^2YZ + 3XY^4Z^2\right)$.

(e) Find the conditional distribution of X and Y, given $Z = z$ and evaluate $E\left(X + Y/Z\right)$.

18. (a) Compute the measures of skewness and Kurtosis of the poisson distribution with mean $\mu$.

(b) Prove that, if the random variable $X$ is $N\left(\mu, \sigma^2\right)$, $\sigma^2 > 0$, then the random variable $V = (X-\mu)^2/\sigma^2$ is $X^2(1)$.

19. Let $Y_1 < Y_2$ denote the order statistics of a random sample of size 2 from $N\left(0, \sigma^2\right)$.

(a) Show that $E(Y_1) = -\sigma/\sqrt{\pi}$.

(b) Find the Covariance of $Y_1$ and $Y_2$.

20. Let the random variable $Y_n$ have a distribution that is $b(n, p)$.

(a) Prove that $Y_n/n$ converges in probability to $p$.

(b) Prove that $1 - Y_n/n$ converges in probability to $1 - p$.

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| F–8461 |
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| 7MMA3E1 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.**

**Third Semester**

**Mathematics**

**Elective – DISCRETE MATHEMATICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define: Semigroup and monoid.

2. What is meant by a monoid homomorphism?

3. Define Ackermann's function.

4. Define a discrete function.

5. What is meant by successor function?

6. Define a regular function. Give an example.

7. When will you say that a poset is said to be a lattice?

8. Give an example of an infinite lattice without a zero and a one.

9. Define the principal disjunctive normal form of p.

10. Draw the NOT-gat and NOR gate.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $(M,*,e)$ be a monoid and $a \in M$. If a is invertible, then prove that its inverse is unique.

Or

(b) Let T be the set of all even integers. Prove that the semigroups $(z, +)$ and $(T, +)$ are isomorphic.

12. (a) Show that

$$1.2+2.3+3.4+\cdots+n.(n+1)=\frac{n(n+1)(n+2)}{3},$$ using mathematical induction.

Or

(b) Find the recurrence relation satisfying $y_n = (A+B_n)4^n$.

13. (a) Solve: $S(k)-3S(k-1)-4S(S-2)=4^k$.

Or

(b) Show that the set of divisors B of a positive integer n is recursive.

14. (a) Let $(L,\leq)$ be a lattice. For any $a,b \in L$, prove the following are equivalent:

(i) $a \leq b$ (ii) $a \vee b = b$ (iii) $a \wedge b = a$

Or

(b) (i) What is meant by lattice homomorphism.

(ii) Draw the Hasse diagram for the lattice $P(\{1,2,3,4\},\subseteq)$.

15. (a) Find the Sum-of-products canonical forms of $f(x_1, x_2, x_3) = ((x_1 + x_2)x_3)\,(x_1 + x_3)$.

Or

(b) Simplify $f = x_1' x_2' + x_1 x_3 x_4 + x_1 x_2 x_4' + x_2' x_3$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) Prove that the property of idempotency is preserved under a semigroup homomorphism.

(b) For any commutative monoid $(M, *)$, prove that the set of idempotent elements of M forms a submonoid.

17. Write the recurrence relation for fibonacci numbers and solve it.

18. Using the generating function solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0$ given $y_1 = 1, y_0 = 2$.

19. Let L be a complemented, distributive lattice and $a, b \in L$. Prove the following are equivalent:

(a) $a \leq b$ (b) $a \wedge b' = 0$ (c) $a' \wedge b = 1$ (d) $b' \leq a'$.

20. For the formula $(P \wedge Q) \vee (\neg R \wedge \neg P)$ draw a corresponding circuit using.

(a) NOT, AND and OR gates.

(b) NAND gates only.

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| 7MMA3E3 |

**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Third Semester**

**Mathematics**

**Elective – AUTOMATA THEORY**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Draw a block diagram of a finite automation.

2. Define a non-deterministic finite automation.

3. Is two grammars of different types can generate the same language? Justify your answer.

4. What is meant by type 3 production?

5. Define transpose set with an example.

6. Define the concatenation with an example.

7. Is $aa^* + bb^*$ is the same as $(a+b)^*$? Justify your answer.

8. Write any two application of pumping lemma.

9. Define a parse tree.

10. Let $G$ be a grammar $S \to SbS \mid a$. Is $G$ ambiguous? Justify your answer.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that for any transition function $\delta$ and for any two input strings $x$ and y, $\delta(q,xy)=\delta(\delta(q,x),y)$.

Or

(b) Construct a deterministic finite automation equivalent to $M=(\{q_0,q_1,q_2,q_3\},\{0,1\}), \quad \delta,q_0,\{q_3\})$ where $\delta$ is given in the following table.

| State/$\Sigma$ | $a$ | $b$ |
|---|---|---|
| $\rightarrow q_0$ | $q_{0,}\, q_1$ | $q_0$ |
| $q_1$ | $q_2$ | $q_1$ |
| $q_2$ | $q_3$ | $q_3$ |
| $q_3$ | | $q_2$ |

12. (a) Let $G=(\{S,C\},\{a,b\},P,S)$, where P consists of $S\rightarrow aCa$, $C\rightarrow aCa\,|\,b$. Find $L(G)$.

Or

(b) Prove that the set of all non-palindromes over $\{a,b\}$ is a context-free language.

13. (a) Prove that there exists a recursive set which is not a context-sensitive language over $\{0,1\}$.

Or

(b) Let $\mathcal{L}_{cf1}$ denote the family of type of languages. Prove that the class $\mathcal{L}_{cf1}$ is closed under transpose operation.

14. (a) State and prove the Arden's theorem.

Or

(b) Show that the set $L = \{a^{i^2} / i \geq 1\}$ is not regular.

15. (a) Consider $G$ whose productions are $S \rightarrow aAS / a$, $A \rightarrow SbA / SS / ba$. Show that $S \overset{*}{\Rightarrow} aabbaa$ and construct a derivation tree whose yield is *aabbaa*.

Or

(b) Find a grammar in Chomsky normal form equivalent to $S \rightarrow aAbB$, $A \rightarrow aA / a$, $B \rightarrow bB / b$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) Enumerate the characteristics of automation.

(b) Construct a deterministic automation equivalent to $M = (\{q_0, q_1\}, \{0,1\}), \delta, q_0, \{q_0\})$, where $\delta$ is defined by its state table is given in the following :

| State/$\Sigma$ | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $q_0$ | $q_1$ |
| $q_1$ | $q_1$ | $q_0, q_1$ |

17. Prove that every monotonic grammar $G$ is equivalent to a type 1 grammar.

18. Show that a context-sensitive language is recursive.

19. (a) If $L$ is a regular set over $\Sigma$, then prove that $\Sigma^* - L$ is also regular over $\Sigma$.

(b) If $X$ and $Y$ are regular sets over $\Sigma$, then prove that $X \cap Y$ is also regular over $\Sigma$.

20. Let G be $S \to AB$, $A \to a$, $B \to c/b$, $C \to D$, $D \to E$ and $E \to a$. Eliminate unit productions and get an equivalent grammar.

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Third Semester**

**Mathematics**

**Elective — FUZZY MATHEMATICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down absorption of complement property of crisp set operations.

2. Define membership function.

3. Prove that for all $a, b \in [0, 1]$, $u(a, b) \leq u_{\max}(a, b)$.

4. Define fuzzy complement with an example.

5. What is meant by fuzzy partial ordering?

6. Define max-min composition.

7. Write down the Dempster's rule for combination.

8. Define Sugeno measure.

9. What is meant by measure of confusion?

10. Let $m(\{x_1, x_2\}) = .4$ $m(\{x_3\}) = .1$, $m(\{x_1, x_3\}) = .3$ and $m(\{x_1, x_2, x_3\}) = .2$ be a basic assignment representing a body of evidence with four focal elements. Then calculate $E(m)$.

## Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Show that all $\alpha$-cuts of any fuzzy set $A$ defined on $\mathbb{R}^n$ $(n \geq 1)$ are convex if and only if

$\mu_A[\lambda r + (1-\lambda)s] \geq \min[\mu_A(r), \mu_A(s)]$ for all $r, s \in \mathbb{R}^n$ and all $\lambda \in [0,1]$.

Or

(b) Propose an extension of the standard fuzzy set operations (min, max, $1-a$) to interval valued fuzzy sets.

12. (a) Prove that, if $C$ is a continuous fuzzy complement, then $C$ has a unique equilibrium.

Or

(b) Prove that the sugeno complements are monotonic nonincreasing for all $\lambda \in (-1, \infty)$.

13. (a) Given a crisp equivalence relation $R(X, X)$, prove that the family of all equivalence classes of $R(X, X)$ forms a partition on $X$.

Or

(b) Solve the following fuzzy relation equations

$$P_0 \begin{pmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{pmatrix} = [.6 \quad .6 \quad .5]$$

14. (a) Let $X = \{a, b, c, d\}$. Given the basic assignment $m(\{a, b, c\}) = .5$ $m(\{a, b, d\}) = .2$ and $m(X) = .3$, determine the corresponding belief and plausibility measures.

Or

(b) Show that every possibility measure $\pi$ on $\mathcal{P}(X)$ can be uniquely determined by a possibility distribution function $r : X \to [0, 1]$ via the formula $\pi(a) = \max_{x \in A} r(x)$ for each $A \in \mathcal{P}(X)$.

15. (a) Prove that $H(X,Y) \le H(X) + H(Y)$.

Or

(b) Let $m_X$ and $m_Y$ be marginal basic assignments on set $X$ and $Y$, respectively and let $m$ be a joint basis assignment on $X \times Y$ such that $m(A \times B) = m_X(A) \cdot M_Y(B)$ for all $A \in \mathcal{P}(X)$ and $B \in \mathcal{P}(Y)$. Then prove that $E(m) = E(m_X) + E(m_Y)$

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Consider the fuzzy sets $A$,$B$ and $C$ defined on the interval $x = [0, 10]$ of real numbers by the membership grade function

$$\mu_A(x) = \frac{x}{x+2}, \ \mu_B(x) = 2^{-x}, \ \mu_C(x) = \frac{1}{1 + 10(x-2)^2}.$$

Determine the mathematical formulas and graphs of the membership grade functions of the each of the following

(a) $\overline{A}, \overline{B}, \overline{C}$

(b) $A \cup B, A \cup C\ B \cup C$

(c) $A \cup B \cup C, A \cap B \cap C$

(d) $A \cap \overline{C}, \overline{\overline{B} \cap C}, \overline{A \cup C}$.

17. Show that fuzzy set operations of union, intersection, and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idempotent (or) distributive.

18. Prove the following propositions :

(a) When $R(X,X)$ is strictly antisymmetric crisp relation, then $R \cap R^{-1} = \phi$

(b) When $R(X,X)$ is max-min transitive then $R \circ R \subseteq R$.

19. Let $X = \{a, b, c, d, e, f, g\}$ and $Y = N_7$. Using joint probability distribution on $X \times Y$, given interms of the matrix

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} .08 & 0 & .02 & 0 & 0 & .01 & 0 \\ 0 & .05 & 0 & 0 & .05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .03 \\ .03 & 0 & 0 & .3 & 0 & 0 & 0 \\ 0 & 0 & .01 & .01 & .2 & .03 & 0 \\ 0 & .05 & 0 & 0 & 0 & .1 & 0 \\ 0 & 0 & 0 & .02 & 0 & .01 & 0 \end{pmatrix} \end{array}$$

Determine :

(a) Marginal probabilities

(b) Both conditional probabilities

(c) Hypothetical joint probability distribution based on the assumption of non-interaction

20. State and prove Gibbs theorem.

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Third Semester**

**Mathematics**

**Elective – STOCHASTIC PROCESSES**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a state space with an example.

2. Define a Covariance stationary process.

3. What is meant by the first passage time distribution?

4. What is Markov chains?

5. Define ergodicity.

6. Define a residual time of an interval.

7. When we say that the process is renewal?

8. What is immigration-death process?

9. Write down two random variables in renewal theory.

10. When the modified process reduces to the ordinary renewal process?

**Part B** $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Consider the process $\{X(t), t \in T\}$ whose probability distribution, under a certain condition, is given by

$$\Pr\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1, 2, \ldots \\ = \dfrac{at}{1+at}, n = 0 \end{cases}$$

Find $E\{X(t)\}$ and Var $\{X(t)\}$.

Or

(b) Show that a martingale has a constant mean.

12. (a) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states $0, 1, 2$ and with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution.

Pr $\{X_0 = i\} = \frac{1}{3}, i = 0, 1, 2$ Find

Pr $\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$.

Or

(b) Consider the Markov chain with t.p.m

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & \tfrac{1}{2} & 0 & \tfrac{1}{2} \\ 2 & 0 & 1 & 0 \end{array}$$

Show that the states of the chain are periodic and persistent non-null.

13. (a) State and prove additive property of poisson process.

Or

(b) Show that the interval between two successive occurrences of a poisson process $\{N(t); t \geq 0\}$ having parameter $\lambda$ has a negative exponential distribution with mean $\frac{1}{\lambda}$.

14. (a) Derive the equation of the birth process.

Or

(b) Explain Birth-immigration process.

15. (a) State and prove renewal theorem.

Or

(b) Let $\{X_i\}$ be a sequence of independent random variables, having the some expectation and let N be a stopping time, for $\{X_i\}$ and $E(N) < \infty$, then prove that $E\left\{\sum_{i=1}^{N} X_i\right\} = E(X_i)\,E(N)$.

## Part C (3 × 10 = 30)

Answer any **three** questions.

16. Three players A,B and C, with initially a, b and c units of money (chips) respectively play a game with the following rule. One of the players chosen at random is to give away 1 unit and one of the other two players chosen at random is to receive it. The game is continued till one of the players has all the (a+b+c) units (and the other two are eliminated having no unit of money left). Find the expected duration of the game (number of plays till one player has all the a+b+c units).

17. Derive the chapman –kolmogorov equation, which is satisfied by the transition probabilities of a Markov chain.

18. Show that, under the postulates for poisson process, N(t) follows poisson distribution with mean $\lambda t$.

19. Show that, in a finite irreducible Markov chain, all states are non-null persistent.

20. Prove that for all

$$t > 0, \text{ and } n = 0, 1, 2 \ldots \Pr\left\{N_e(t) \geq n\right\} \geq \Pr\left\{N(t) \geq n\right\}$$

$$\text{iff } \frac{\int_t^{\infty} \{1 - F(x)\} dx}{1 - F(t)} \leq \mu.$$

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022.**

**Third Semester**

**Mathematics**

**Elective — COMBINATORIAL MATHEMATICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a stirling number of the second kind.

2. Write down the partitions of the integer 5 into parts that are powers of 2.

3. Define a pattern with an example.

4. Give an example of a recurrence relation with two indices.

5. What is meant by a rook polynomial? Give an example.

6. Give an example for derangement of integers.

7. Define a one-to-one and onto functions.

8. Define a store enumerator in R.

9. Define a complete block design of $X = \{x_1, x_2, ..., x_v\}$ objects.

10. Define an orthogonal Latin squares.

## Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that the number of ways of selecting an even number of objects is equal to the number of ways of selecting an odd number of objects from $n$ distinct objects.

Or

(b) Find the ordinary enumerator for the selection of $r$ objects out the $n$ objects $(r \geq n)$, with unlimited repetitions but with each object included in each selection.

12. (a) Let there be $n$ ovals drawn on the plane. If an oval intersects each of the other ovals at exactly two points and no three ovals meet at the same point, into how many regions do these ovals divide the plane?

Or

(b) Find the number of n-digit binary sequences that have the pattern 010 occurring at the $n^{th}$ digit.

13. (a) Find the number of integers between 1 and 250 that are not divisible by 2 nor by 7 but are divisible by 5.

Or

(b) Consider the permutations of the $n$ integers 1, 2,..., n. Find the number of permutations in which no two adjacent integers are consecutive integers.

14. (a) Find the number of distinct bracelets of five beads made up of yellow, blue and white beads.

Or

(b) In how many ways can five books, two of which are the same, be distributed to four children, if among them there is a set of identical twins?

15. (a) If a set of $r$ orthogonal Latin squares of order $n$ and a set of $r$ orthogonal Latin squares of order n' exist, then prove that there exists a set of $r$ Latin squares of order $nn'$.

Or

(b) For $a(b, v\, r, k, \lambda)$ - configuration, prove that $QQ^T = (r - \lambda)\, I + \lambda J$, where I is the $v \times v$ identity matrix, and J is the $v \times v$ matrix in which all the entries are 1's.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Evaluate the sum $1^2 + 2^2 + 3^2 + \ldots + r^2$.

17. Find the number of ways to parenthesize the expression $w_1 + w_2 + \ldots + w_{n-1} + w_n$ so that only two terms will be added at one time.

18. Derive the principle of inclusion and exclusion.

19. State and prove Polya's fundamental theorem.

20. For $n \geq 3$ and $n = P^{\alpha}$, where P is a prime number and $\alpha$ is a positive integer, prove that there exists a set of $n-1$ orthogonal Latin squares of order.

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Fourth Semester**

**Mathematics**

**OPERATIONS RESEARCH**

**(CBCS – 2017 onwards)**

Time : 3 Hours　　　　Maximum : 75 Marks

**Part A**　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define a cycle. Give an example.
2. Classify of the activities of the project as critical and noncritical.
3. What is meant by setup cost?
4. Write down the general assumptions of the no-setup model.
5. What is role of queue size?
6. What is meant by jockey?
7. Define State dependent. Give an example.
8. Draw the Transition-rate diagram.
9. Illustration of the general step of the dichotomous and golden section search methods.
10. Define the general constrained nonlinear programming problem.

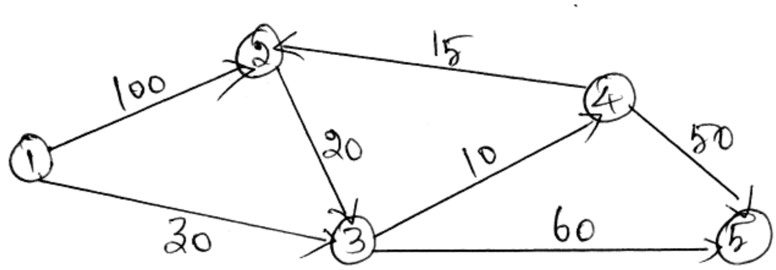
## Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain minimal spanning tree algorithm.

Or

(b) Use Dijkstra's algorithm to find the shortest route between node 1 and every other node in the following network.




12. (a) Metal co produces draft deflectors for use in home fire places during the months of December to March. The demand starts slow, peaks in the middle of the season and tapers off toward the end. Because of the popularity of the product, Metal co may use overtime to satisfy the demand. Find the optimal solution for the following table provides the production capacities and the demands for the four winter months.

| | Capacity | | |
|---|---|---|---|
| Month | Regular (units) | Overtime (units) | Demand (units) |
| 1 | 90 | 50 | 100 |
| 2 | 100 | 60 | 190 |

| | Capacity | | |
|---|---|---|---|
| Month | Regular (units) | Overtime (units) | Demand (units) |
| 3 | 120 | 80 | 210 |
| 4 | 110 | 70 | 160 |

Unit production cost in any is \$6 during regular time and \$9 during overtime. Holding cost per unit per month is \$10.

Or

(b) Explain Silver-meal Heuristic.

13. (a) Derive Pure birth model.

Or

(b) The Florist section in a grocery store stocks 18 dozen roses at the beginning of each week. On the average, the florist sells 3 dozens a day, but the actual demand follows a Poisson distribution. Whenever the stock level reaches 5 dozens, a new order of 18 new dozens is placed for delivery at the beginning of the following week. Because of the nature of the item, all roses left at the end of the week are disposed of. Determine the following :

(i) The probability of placing an order in any one day of the week.

(ii) The average number of dozen roses that will be discarded at the end of the week.

14. (a) Automata car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The time for washing and cleaning a car is exponential, with a mean of 10 minutes. Cars that cannot park in the lot can wait in the street bordering the wash facility. This means that for all practical purposes, there is no limit on the size of the system. The manager of the facility wants to determine the size of the parking lot.

Or

(b) Explain $(M/M/C):(GD/\infty/\infty)$ queuing model.

15. (a) Solve the following problem using Golden section method

$$\text{Maximize } f(x) = \begin{cases} 3x, & 0 \le x \le 2 \\ \frac{1}{3}(-x+20), & 2 \le x \le 3 \end{cases}$$

Given the maximum value of $f(x)$ occurs at $x = 2$ and $\Delta = 0.1$.

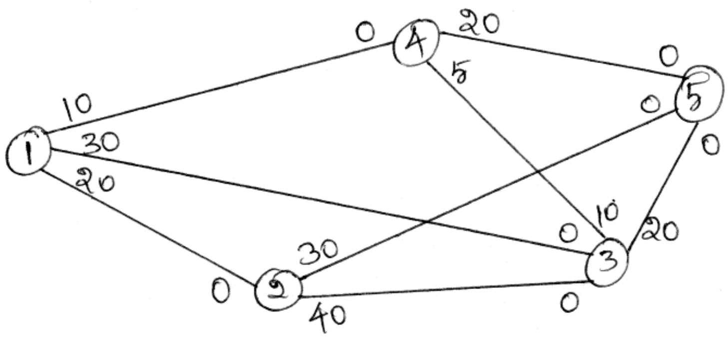
Or

(b) Explain Steepest ascent method.

## Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the maximal flow in the following network




17. Neon lights on the U of A campus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs \$100 to initiate a purchase order. 4 neon light kept in storage is estimated to cost about \$0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

18. Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following :

   (a) The average number of births per year

   (b) The probability that no births will occur in any one day

   (c) The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3 hours period.

19. B & K Groceries operates with three checkout counters. The manager uses the following schedule to determine the number of counters in operation depending on the number of customers in the store :

| No. of customers in the store | No. of counters in operation |
|---|---|
| 1 to 3 | 1 |
| 4 to 6 | 2 |
| More than 6 | 3 |

Customers arrive in the counter area according to a Poisson distribution with a mean rate of 10 customers per hour. The average checkout time per customer is exponential with mean 12 minutes. Determine the steady-state probability in of $n$ customers in the checkout area.

20. Solve the following non-linear programming problem using separable programming method :

Maximize $Z = x_1 + x_2^4$

subject to $3x_1 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0.$

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Fourth Semester**

**Mathematics**

**TOPOLOGY – II**

**(CBCS – 2017 onwards)**

Time : 3 Hours                                        Maximum : 75 Marks

**Part A**                    $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by one-point compactification?

2. Show that the rationals $Q$ are not locally compact.

3. When will you say that two compactification are equivalent?

4. Under what condition does a metrizable space have a metrizable compactification?

5. Give an example of a collection of sets G that is not locally finite such that the collection $B = \{\overline{A} / A \in G\}$ is locally finite.

6. State Bing metrization theorem.

7. Consider the subset $R^{\infty}$ of $R^{w}$ consisting of sequences that are eventually zero. Is $R^{\infty}$ complete in the uniform metric?

8. What is meant by topology of pointwise convergence?

9. Define Baire space.

10. Given $n$ and $\in$, define a continuous function $f : I \to R$ such that $f \in U_n$ and $|f(x)| \leq \in$ for all $x$.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X$ be locally compact Hausdorff; let $Y$ be a subspace of $X$. If $Y$ is closed in $X$ or open in $X$, then prove that $Y$ is locally compact.

Or

(b) Show that if $G$ is a locally compact topological group and $H$ is a subgroup, then $G/H$ is locally compact.

12. (a) Let $X$ be completely regular; let $\beta(X)$ be its stone - Čech compactification. Then prove that every bounded continuous real - valued function on $X$ can be uniquely extended to a continuous real - valued function on $\beta(x)$.

Or

(b) Show that every locally compact Hausdorff space is completely regular.

13. (a) Let $X$ be a metrizable space. Then prove that X has a basis that is countably locally finite.

Or

(b) Find a space that has a countably locally finite basis but does not have a countable basis.

14. (a) Let $(X, d)$ be a metric space. Show that there is an isometric imbedding of $X$ into a complete metric space.

Or

(b) If $X$ is locally compact, or if $X$ satisfies the first countability axiom, then prove that $X$ is compactly generated.

15. (a) Show that, if $X$ is a compact Hausdorff space, or a complete metric space, then $X$ is a Baire space.

Or

(b) Let X be a space and let $(Y, d)$ be a metric space, prove that the space $\mathcal{C}(X, Y)$, the compact - open topology and the topology of compact convergence coincide.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Tychonoff theorem.

17. Let X be discrete; consider $\beta(X)$.

(a) Show that if $A \subset X_1$ then $\overline{A}$ and $\overline{X - A}$ are disjoint.

(b) Show that if $U$ is open in $\beta(X)$, then $\overline{U}$ is open in $\beta(X)$.

(c) Show that $\beta(X)$ is totally disconnected.

18. Let $X$ be a regular space with a basis $B$ that is countably locally finite. Then prove that $X$ is metrizable.

19. Prove that a metric space $(X,d)$ is compact if and only if it is complete and totally bounded.

20. State and prove Ascoli's theorem.

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**M.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**Fourth Semester**

**Mathematics**

***Elective* — ADVANCED STATISTICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by an invariance property of a maximum likelihood estimator?

2. Define a power function of a test.

3. Define a complete family of Probability density functions.

4. State a principle of selecting a best decision function.

5. Define a Baye's solution.

6. Define an efficient estimator of a parameter.

7. What is meant by likelihood ratio test?

8. Define a noncentral $t$-distribution with $r$ degrees of freedom.

9. Define a noncentral Chi-square distribution.

10. What is meant by a test of independence?

## Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X_1, X_2, ... X_n$ denote a random sample from the distribution with p.d.f.

$$f(x) = \theta^x (1-\theta)^{1-x}, \; x = 0, 1$$
$$= 0, \qquad \textit{elsewhere}$$

where $0 \le \theta \le 1$.

Find the m.l.e. $\hat{\theta}$ of $\theta$.

Or

(b) Let two independent random samples, each of size 10, from two normal distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8, s_1^2 = 8.64, \overline{y} = 5.6, s_2^2 = 7.88$. Find a 95 percent confidence interval for $\mu_1 - \mu_2$.

12. (a) If $az^2 + bz + c = 0$ for more than two values of $z$, then $a = b = c = 0$. Use this result to show that the family $(b(2, \theta) : 0 < \theta < 1\}$ is complete.

Or

(b) Let $X_1, X_2, ..., X_n, n > 2$, be a random sample from the binomial distribution $b(1, \theta)$. Show that $Y_1 = X_1 + X_2 + ... + X_n$ is a complete sufficient statistic for $\theta$.

13. (a) Let $X$ be $N(\theta, \sigma^2)$, where $-\infty < \theta < \infty$ and $\sigma^2$ is known. Compute $I(\theta)$.

Or

(b) Prove that $\overline{X}$, the mean of a random sample of size $n$ from a distribution that is $N(\theta, \sigma^2)$, $-\infty < \theta < \infty$, is, for every known $\sigma^2 > 0$, an efficient estimator of $\theta$.

14. (a) Let $X_1, X_2, ..., X_{25}$ denote a random sample of size 25 from a normal distribution $N(\theta, 100)$. Find a uniformly most powerful critical region of size $\sigma = 0.10$ for taking $H_0 : \theta = 75$ against $H_1 : \theta > 75$.

Or

(b) Explain Wald's sequential probability ratio test.

15. (a) Explain a noncentral $F$-variable with $r_1$ and $r_2$ degrees of freedom and with non centrality parameter $\theta$.

Or

(b) Compute the mean and variance of a random variable that is $\chi^2 \, (r, \theta)$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Let the following sets be defined : $A_1 = \{x : -\infty < x \leq 0\}$, $A_i = \{x : i - 2 < x \leq i - 1\}$, $i$ = 2,...,7 and $A_8 = \{x : 6 < x < \infty\}$. A certain hypothesis assigns probabilities $P_{i_0}$ to these sets $A_i$ in accordance with $P_{i_0} = \int_{A_i} \frac{1}{2\sqrt{2\pi}}$ $\exp\left[-\frac{(x-3)^2}{2(4)}\right] dx, i = 1, 2, ..., 7, 8$.

This hypothesis is to be tested, at the 5 percent level of significance by a chi-square test. If the observed frequencies of the sets $A_i = i = 1, 2, ..., 8$ are respectively, 60, 96, 140, 210, 172, 160, 88 and 74, would $H_0$ be accepted at the 5 percent level of significance?

17. State and prove the factorization theorem of Neyman.

18. Derive Fisher information $I(\theta) = \int_{-\infty}^{\infty} \left[ \frac{\partial \inf(:\theta)}{\partial \theta} \right]^2 f(x:\theta)\, dx$.

19. State and prove Neyman-Pearson theorem.

20. If $A_1, A_2, \ldots, A_k$ are events. Prove, by induction, Boole's inequality $P(A_1 \cup A_2 \cup \ldots \cup A_k) \leq \sum_{i=1}^{k} P(A_i)$. Also prove that $P(A_1^* \cap A_2^* \cap \ldots \cap A_k^*) \geq 1 - \sum_{i=1}^{k} P(A_i)$.